Marwari College, Darbhanga

D-ll Physics Hons.

Paper-III, Group - B

Topic: (1) Lorentz Gauge

(2) Coulomb Gauge

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Lorentz Gauge:

The maxwell's field equation in terms of e.m. potentials are,

$$\nabla^2 \vec{A} - \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t^2} - \nabla\left(\vec{\nabla}\cdot\vec{A} + \mu\epsilon \cdot \frac{\partial V}{\partial t}\right) = -\mu\vec{J} \quad \text{—①}$$

$$\nabla^2 V - \mu\epsilon \cdot \frac{\partial^2 \phi}{\partial t^2} + \frac{\partial}{\partial t}\left(\nabla\cdot\vec{A} + \mu\epsilon \cdot \frac{\partial V}{\partial t}\right) = -\frac{\rho}{\epsilon}$$

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②

eq^n ① and ②, will be much more simplified if we choose

$$\nabla\cdot\vec{A} + \mu\epsilon \cdot \frac{\partial V}{\partial t} = 0 \quad \text{—③}$$

This requirement is called the Lorentz Condition and when the vector and scalar potential satisfy it the gauge is known as Lorentz gauge.

So with Lorentz Condition, field eq^n reduces to,

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$$\nabla^2 \vec{A} - \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J} \quad \text{——} (4)$$

and, $$\nabla^2 V - \mu\epsilon \frac{\partial^2 \phi}{\partial t^2} = -\frac{\rho}{\epsilon} \quad \text{——} (5)$$

But as, $\mu\epsilon = \frac{1}{v^2}$

So eqn (4) and (5) Can be written as,

$$\boxed{\begin{aligned} \Box^2 \vec{A} &= -\mu \vec{J} \\ \Box^2 V &= -\frac{\rho}{\epsilon} \end{aligned}} \quad \begin{aligned} &\text{——} (6) \\ &\text{——} (7) \end{aligned}$$

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where, $$\Box^2 = \nabla^2 - \frac{1}{v^2} \cdot \frac{\partial^2}{\partial t^2}$$

eq. (6) and (7) are inhomogeneous wave eqs. and are known as D'Alembertian eqs.

In order to determine the requirement that Lorentz Condition places on Λ,

we substitute A' and V' in eqn
(3), i.e

$$\nabla\cdot(\vec{A'} - \nabla\Lambda) + \mu\epsilon\,\frac{\partial}{\partial t}\left(V' + \frac{\partial\Lambda}{\partial t}\right) = 0$$

i.e. $$\nabla\cdot\vec{A'} + \mu\epsilon\,\frac{\partial V'}{\partial t} = \nabla^2\Lambda - \epsilon\mu\frac{\partial^2\Lambda}{\partial t^2}$$

So, $\vec{A'}$ and V' will also satisfy eqn (3) i.e. Lorentz condition provided that

$$\nabla^2\Lambda - \epsilon\mu\,\frac{\partial^2\Lambda}{\partial t^2} = 0$$

i.e. $$\Box^2\Lambda = 0$$

i.e., Lorentz condition is invariant under those gauge transformation for which the gauge functions are solutions of the homogeneous wave eqs.

Coulomb Gauge:

Field eqs. in terms of e.m. potentials i.e.

$$\nabla^2 \vec{A} - \mu\epsilon \frac{\partial^2 \vec{A}}{\partial t^2} - \nabla\left(\vec{\nabla}\cdot\vec{A} + \mu\epsilon \frac{\partial V}{\partial t}\right) = -\mu\vec{J} \quad \text{--- (1)}$$

and, $$\nabla^2 V - \mu\epsilon \frac{\partial^2 V}{\partial t^2} + \frac{\partial}{\partial t}\left(\vec{\nabla}\cdot\vec{A} + \mu\epsilon \frac{\partial V}{\partial t}\right) = -\frac{\rho}{\epsilon}$$

$$\cancel{\text{or, } \nabla^2 V + \frac{\partial}{\partial t}(\nabla\cdot\vec{A}) = -\frac{\rho}{\epsilon}} \quad \text{--- (2)}$$

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If we assume,

$$\boxed{\nabla\cdot\vec{A} = 0} \quad \text{--- (3)}$$

eq^n (2) reduces to Poisson's eq^n.

$$\nabla^2 V = -\frac{\rho}{\epsilon} \quad \text{--- (4)}$$

whose solution is,

$$V(r,t) = \frac{1}{4\pi\epsilon}\int \frac{\rho(r',t)^*}{R}\, d\tau' \quad \text{--- (5)}$$

the scalar potential is the coulombian potential due to charge $\rho(x', y', z', t)$.
This is the origin of the name Coulomb gauge.

using eqn ③, eqn ① reduces to

$$\nabla^2 \vec{A} - \frac{1}{v^2} \cdot \frac{\partial^2 \vec{A}}{\partial t^2} = -\mu \vec{J} + \mu \epsilon \nabla \cdot \left(\frac{\partial V}{\partial t}\right) \quad (6)$$